Midsegment Theorem

Goals
- Identify the midsegments of a triangle.
- Use properties of midsegments of a triangle.

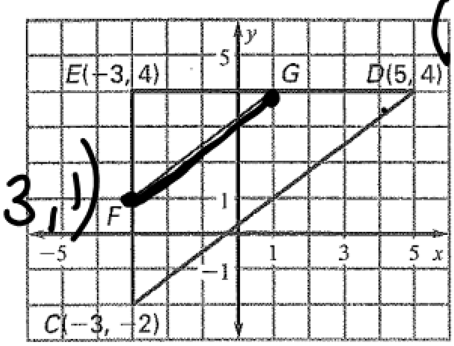
VOCABULARY

Midsegment of a triangle
the segment that connects the midpoints of two sides of a $\triangle$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Example 1 *Using Midsegments*

Show that the midsegment $\overline{FG}$ is parallel to side $\overline{CD}$ and is half as long as $\overline{CD}$.



(1,4)

(−3,1)

Solution

Use the Midpoint Formula to find the coordinates of F and G.

$$F = \left(\frac{-3+-3}{2}, \frac{4+(-2)}{2}\right) = (-3, 1)$$

$$G = \left(\frac{-3+5}{2}, \frac{4+4}{2}\right) = (1, 4)$$

Next, find the slopes of $\overline{CD}$ and $\overline{FG}$.

Remember: The slope m of the line passing through (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\text{Slope of } \overline{CD} = \frac{-2-4}{-3-5} = \frac{-6}{-8} = \frac{3}{4}$$

$$\text{Slope of } \overline{FG} = \frac{4-1}{1-(-3)} = \frac{3}{4}$$

$$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

▶ Because the slopes are equal, $\overline{FG}$ is parallel to $\overline{CD}$.

Next, find the lengths of $\overline{CD}$ and $\overline{FG}$.

Use the Distance Formula to find the lengths.

$$CD = \sqrt{[-3 - (5)]^2 + [-2 - (4)]^2} = \sqrt{100} = 10$$

$(-8)^2$ $(-6)^2$

$$FG = \sqrt{[4 - (1)]^2 + (1 - -3)^2} = \sqrt{25} = 5$$

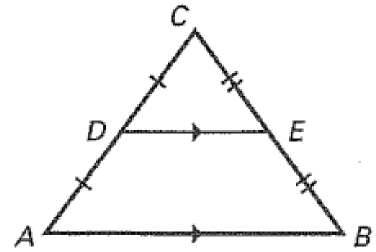
3^2 4^2

▶ Because $\frac{FG}{CD} = \frac{5}{10} = \frac{1}{2}$, $\overline{FG}$ is half as long as $\overline{CD}$.

THEOREM 5.9: MIDSEGMENT THEOREM

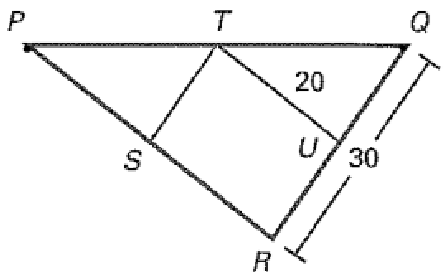
The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.

$\overline{DE} \parallel \overline{AB}$ and $DE = \frac{1}{2} AB$



Example 2 *Using the Midsegment Theorem*

$\overline{ST}$ and $\overline{TU}$ are midsegments of $\triangle PQR$. Find PR and ST.

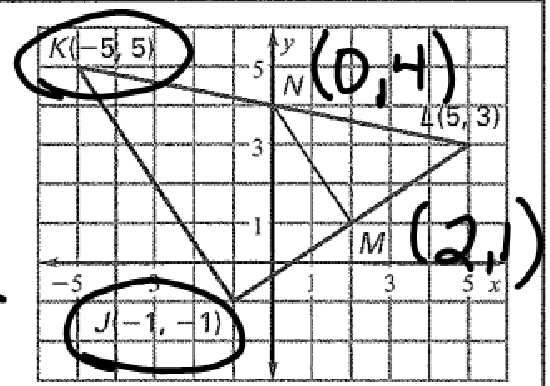


Solution

$PR = 2(TU) = 2(20) = 40$

$ST = \frac{1}{2}(RQ) = \frac{1}{2}(30) = 15$

1. Show that the midsegment $\overline{MN}$ is parallel to side $\overline{JK}$ and is half as long as $\overline{JK}$.



$MN \rightarrow \frac{-3}{2}$ $JK \rightarrow \frac{-6}{4} = \frac{-3}{2}$

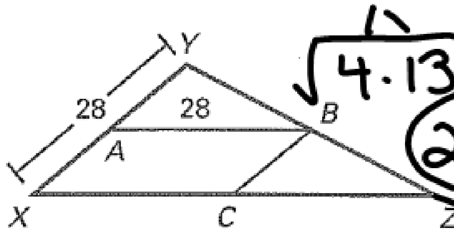
$MN = \sqrt{(2-0)^2 + (1-4)^2}$
$= \sqrt{4+9} = \sqrt{13}$

$JK = \sqrt{(-1-(-5))^2 + (-1-5)^2} = \sqrt{16+36} = \sqrt{52}$

$\sqrt{4 \cdot 13}$

$2\sqrt{13}$

2. $\overline{AB}$ and $\overline{BC}$ are midsegments of $\triangle XYZ$. Find XZ and BC.



$XZ = 2(AB) = 2(28) = 56$

$BC = \frac{1}{2}(XY) = \frac{1}{2}(28) = 14$